

# Cosmic Superstrings and Primordial Magnetogenesis

Anne-Christine Davis<sup>1,\*</sup> and Konstantinos Dimopoulos<sup>2,†</sup>

<sup>1</sup>*Department of Applied Mathematics and Theoretical Physics,  
Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK*

<sup>2</sup>*Department of Physics, Lancaster University, Lancaster LA1 4YB, UK*

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Cosmic superstrings are produced at the end of brane inflation. Their properties are similar to cosmic strings arising in grand unified theories. Like cosmic strings they can give rise to a primordial magnetic field, as a result of vortical motions stirred in the ionised plasma by the gravitational pull of moving string segments. The resulting magnetic field is both strong enough and coherent enough to seed the galactic dynamo and explain the observed magnetic fields of the galaxies.

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## I. INTRODUCTION

Cosmic superstrings have received a lot of interest recently due to developments in fundamental string theory. They arise naturally in models of brane inflation and have characteristic differences with ordinary cosmic strings, which could provide a window into string theory. There is also the distinct possibility that they could solve some long-standing problems in cosmology and astroparticle physics.

For example, magnetic fields pervade most astrophysical objects [1, 2], but their origin is still elusive. In the last decade a number of attempts were made to explain the observed magnetic fields of the galaxies, none of which has been conclusive. Many authors have considered that these magnetic fields originate from the Early Universe and are truly primordial. Since a large scale primordial magnetic field (PMF) cannot be generated in thermal equilibrium (because it breaks isotropy), research has been focused in magnetogenesis mechanisms either during phase transitions or from inflation. Phase transitions occur very early in the history of the Universe. Consequently, any PMF generation creates highly incoherent magnetic fields [3] that cannot give rise to the magnetic fields of the galaxies (unless one considers inverse cascade mechanisms [4]). On the other hand, due to the conformal invariance of electromagnetism, generating a PMF during inflation substantially dilutes its strength down to insignificant values [5] (see however [6]). An extensive review of the literature on PMFs can be found in [7] (see also references in [6]).

Recent developments in string theory offer another possibility that PMFs could have a fundamental origin. The motion of a network of cosmic strings can result in a primordial magnetic field which is strong enough to seed the galactic dynamo [8, 9, 10, 11, 12]. However, cosmic strings arising in grand unified theories seem to be at variance with the observations of the CMB. Models of brane

inflation predict the formation of cosmic superstrings at the end of inflation [13, 14]. Such cosmic superstrings have a lower string tension than those arising in grand unified theories, so they evade the CMB limits on cosmic strings (for reviews see [15, 16]). Consequently, a network of cosmic superstrings could produce a viable primordial magnetic field and still be consistent with other cosmological observations.

In this paper we investigate this possibility, presenting two mechanisms for the production of a primordial magnetic field from a network of cosmic superstrings. In Sec. II we discuss cosmic superstrings and their characteristics; their tension and intercommutation probability. In Sec. III we present the magnetogenesis mechanism, based on the effect of a network of cosmic superstrings onto ionised plasma. We consider two realisations of this mechanism; one generating a PMF inside the string wakes and the other over inter-string distances. We also consider the cases of wiggly strings or current carrying strings. Finally, in Sec. IV we discuss our results and present our conclusions. Throughout the paper we use natural units, such that  $c = \hbar = 1$ . The signature of the spacetime metric is taken to be  $(-, +, +, +)$ .

## II. COSMIC SUPERSTRINGS

There has been a resurgence of interest in cosmic strings arising from recent results in fundamental string theory. Indeed, they are predicted to arise in models of brane inflation where an extra brane and anti-brane annihilate to produce lower dimensional branes, with the inter-brane distance playing the role of the inflaton. In this picture D-strings, or  $D1$  branes are formed generically [13]. Similarly fundamental strings, or F-strings, can also arise [14] and, in certain classes of models, axionic local strings [17].

In models of brane inflation the extra brane and anti-brane are localised at the bottom of a throat in the compact dimensions. Consequentially, D-strings and F-strings are also formed in the throat. Since space-time is highly warped in the throat this results in the string tension of the D- and F-strings being less than the fundamental

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\*Electronic address: acd@damtp.cam.ac.uk

†Electronic address: k.dimopoulos1@lancaster.ac.uk

scale,

$$\mu = e^{-A(y)}\mu_0, \quad (1)$$

where  $A(y)$  is the warp factor with  $y$  referring to the compact dimensions and  $\mu_0$  is the fundamental scale. Estimates give the range to be between  $10^{-12} \leq G\mu \leq 10^{-6}$  depending on details of the theory (see [15, 16] for a review).

The gravitational effects of cosmic superstrings will be similar to those of the usual cosmic strings and they will be subject to the same constraints. For example, we know that cosmic strings are not the primary source of structure formation, resulting in a constraint of  $G\mu < 3.3 \times 10^{-7}$  arising from the WMAP data [19]. Similarly the regularity of the pulsar timings results in a constraint on gravitational waves emitted by cosmic strings, corresponding to  $G\mu < 10^{-7}$  (see for example [18] and references therein). Hence we arrive at the range

$$10^{-12} \leq G\mu < 10^{-7} \quad (2)$$

Supersymmetric theories give rise to two sorts of strings, called D-term or F-term strings [20], where the D and F refer to the type of potential required to break the symmetry. A natural question to ask is whether these cosmic strings are related to the D- and F-strings discussed above. A recent analysis of supersymmetric theories with a D-term suggests that D-term cosmic strings may well be D-strings [21]. It is then possible that D-strings are current-carrying via fermion zero modes since it was shown that fermion zero modes survive supersymmetry breaking for D-term strings [22].

The cosmology of D-strings (and F-strings) is a little different from that of ordinary cosmic strings. For ordinary cosmic strings, the probability of intercommutation is  $P \simeq 1$ . This is not the case for D-strings since they can ‘miss’ each other in the compact dimension, whilst for F-strings intercommutation is a quantum mechanical process. The probability of intercommuting has been estimated to be between  $10^{-1} \leq P \leq 1$  for D-strings and  $10^{-3} \leq P \leq 1$  for F-strings [15]. Similarly the probability of a string self-intersecting is reduced. This means that a network of such strings could look different from that of cosmic strings. There are suggestions that such a network would be denser, with the distance between strings related to  $P$ , and slower [23, 24]. It is likely that the net result would be to increase the number of string loops, despite the reduction in string self-intersection. A network of D-strings could also emit exotic particles, such as dilatons [25, 26], as a result of the underlying superstring theory.

The evolution of cosmic superstrings will vary from that of cosmic strings. Usually a cosmic string network reaches a scaling solution. For cosmic superstrings this is still the case however the intercommutation probability comes into the scaling solution. There have been analytic [23, 27] and numerical [24] investigations into the behaviour of the cosmic superstring network, leading to

the conclusion that the correlation length behaves as

$$\xi = P^\beta t, \quad \text{where} \quad \frac{1}{2} \leq \beta \leq 1 \quad (3)$$

Similarly the gravitational radiation emitted from a cosmic superstring network will depend on the parameter  $P$ , which loosens the pulsar constraints on  $G\mu$  discussed above.

Cosmic strings can generate a primordial magnetic field [8, 9, 10, 11, 12]. Similarly we would expect D-strings to give rise to a primordial magnetic field in a similar way to other local cosmic strings. However, there will be distinct differences for D-strings given that their cosmology differs. In some models semi-local strings arise [28, 29], rather than cosmic strings. These are not topologically stable [30], but if they were to live long, they could still contribute in a similar way to cosmic strings, and similarly for the local axionic strings [17].

For F-strings there could still be a primordial magnetic field produced due to the motion of the string through the surrounding plasma. Here, though, the mechanism will be similar to that for global strings.

In the next section we review magnetogenesis mechanisms with cosmic string networks.

### III. MAGNETOGENESIS MECHANISMS

In this section we will study two mechanisms for the generation of a primordial magnetic field (PMF) due to the cosmological effects of a network of cosmic superstrings. Although the existence of a PMF may have many cosmological implications we will focus more on the possibility of explaining the galactic magnetic fields, by triggering the  $\alpha - \Omega$  dynamo mechanism in galaxies after galaxy formation. Such a mechanism requires the presence of a preexisting seed magnetic field in order to operate. This seed field has to satisfy certain requirements in terms of strength and coherence. These are the following.

To successfully trigger the dynamo and explain the galactic magnetic fields the lower bound on the strength of the seed field (in a dark energy dominated Universe) is [31]

$$B_{\text{seed}} \geq 10^{-30} \text{Gauss} \quad (4)$$

Such a seed field is amplified exponentially by the galactic dynamo until it reaches the observed value  $B_{\text{obs}} \sim 10^{-6} \text{Gauss}$ , where it becomes dynamically important (its energy is comparable to the kinetic energy of galactic rotation). At this stage galactic dynamics back-reacts to the dynamo mechanism and stabilises the value of the field. Considering the characteristic timescale for the dynamo operation (the galactic rotation period) a seed field weaker than the bound in Eq. (4) would not have enough time to be amplified up to the observed

value.<sup>1</sup>

Also, for the dynamo action not to be destabilised the seed field has to avoid being too incoherent. Indeed the coherence of the seed field cannot be much smaller than [32]

$$\ell_{\text{seed}} \gtrsim 100 \text{ pc}. \quad (5)$$

In both the mechanisms that we consider the PMF generation is based on the Harrison–Rees mechanism, which is briefly reviewed below.

### A. The Harrison–Rees mechanism

Harrison was the first to consider the generation of a magnetic field by the vortical motions of ionised plasma. He suggested that turbulence in an expanding Universe may generate a magnetic field since the turbulent velocity would be different for the electrons and the much heavier ions [33]. His argument focused in the radiation era and can be sketched as follows.

Consider a rotating volume  $V$  of ionised plasma. Suppose that the angular velocities  $\omega_i$  and  $\omega_e$  of the ion and the electron fluid respectively are uniform inside  $V$ . Then, since  $V \propto a^3$ , we find that

$$\rho_i V = \text{const.} \quad \text{and} \quad \rho_e V^{4/3} = \text{const.}, \quad (6)$$

where  $\rho_i \propto a^{-3}$  is the ion density, which scales like pressureless matter, while  $\rho_e \propto a^{-4}$  is the electron density, which scales as radiation due to the strong coupling between the electrons and the photons, through Thompson scattering. The angular momentum  $\mathcal{I} = \rho \omega V^{5/3}$  of each plasma component has to be conserved. This suggests that

$$\omega_i \propto V^{-2/3} \propto a^{-2} \quad \text{and} \quad \omega_e \propto V^{-1/3} \propto a^{-1}. \quad (7)$$

Thus, the ion fluid spins down faster than the electron-photon gas. Consequently, a circular current is generated, which creates a magnetic field in the volume  $V$ .

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<sup>1</sup> Before the discovery of dark energy the lower bound on  $B_{\text{seed}}$  was much more stringent:  $B_{\text{seed}} \geq 10^{-21} \text{ Gauss}$ . This is easy to understand as follows. The minimum strength of the seed field corresponds to a field which, when amplified by the dynamo from the time of galaxy formation  $t_{\text{gf}}$  until the present time  $t_0$ , just about reaches the observed value of  $1 \mu\text{Gauss}$ . Hence, we have  $1 \mu\text{Gauss} \sim e^N B_{\text{seed}}^{\text{min}}$ , where  $N$  is the number of galactic revolutions since the time of galaxy formation. Now,  $N \sim \Delta t / \tau_g$ , where  $\Delta t = t_0 - t_{\text{gf}} \simeq t_0$  and  $\tau_g$  is the timescale of dynamo amplification (galactic rotation period). Without dark energy  $t_0 \simeq 8.96 \text{ Gyrs}$ , which suggests that the galaxy has rotated about  $N \simeq 35$  times. However, when taking dark energy into account, the age of the Universe is multiplied by a factor  $\frac{1}{\sqrt{\Omega_\Lambda}} \sinh^{-1} \sqrt{\frac{\Omega_\Lambda}{1-\Omega_\Lambda}}$ , which, for  $\Omega_\Lambda \simeq 0.7$ , gives  $t'_0 \simeq 13.7 \text{ Gyrs}$ . Thus, the number of galactic revolutions becomes  $N' = (13.7/8.96)N \simeq 54$ . Hence, the lower bound on the seed field now reads:  $B_{\text{seed}}^{\text{min}} \sim e^{-N'} \times 1 \mu\text{Gauss} \sim 10^{-30} \text{ Gauss}$ .

Rees, however, has shown that expanding volumes of spinning plasma are unstable in the radiation era and decay with cosmic expansion [34]. He suggested instead a different version of vortical magnetic field generation involving Compton scattering of the electrons on the CMB (Compton drag mechanism). This applies after recombination and tends to damp the vortical motions of the electrons in contrast to ions, which remain unaffected. The result is again the generation of circular currents but, this time, it is the electron fluid that slows down.

In both cases, the Maxwell's equations suggest [33]

$$\mathbf{B} \simeq -\frac{m_p}{e} \mathbf{w}, \quad (8)$$

where  $m_p \sim 1 \text{ GeV}$  is the nucleon mass and  $\mathbf{w}$  is the vorticity of the plasma, given by

$$\mathbf{w} = \nabla \times \mathbf{v}_{\text{rot}}, \quad (9)$$

with  $\mathbf{v}_{\text{rot}}$  being the rotational velocity of the spinning plasma. A similar mechanism is presented in [35].

### B. Vortical motions inside the string wakes

Vachaspati and Vilenkin were the first to suggest that vortical motions inside the wakes of cosmic strings can give rise to PMFs [8, 9] (see also [10]). The idea is that the boost generated by the deficit angle of the cosmic string metric may stir vorticity in the matter, which falls into the wake of a travelling string. The vortical motions themselves are generated by the rapidly changing conical metric of the string in the small-scale wiggles, which a long string develops due to self-intersections. The oscillations of the wiggles are expected to generate turbulence in the plasma inside the string wake.

The metric of the (2+1)-dimensional spacetime perpendicular to a straight string is:

$$ds_\perp^2 = -dt^2 + dr^2 + (1 - 8G\mu)r^2 d\phi^2, \quad (10)$$

which describes the space around a cosmic string as Euclidean with a wedge of angular size  $\Delta$  removed, where

$$\Delta = 8\pi G\mu. \quad (11)$$

A test particle at rest with respect to the string experiences no gravitational force but, if the string moves with velocity  $v_s$ , then nearby matter undergoes a boost

$$u = 4\pi G\mu v_s \gamma_s \quad (12)$$

in the direction perpendicular to the motion of the string, where  $\gamma_s = 1/\sqrt{1-v_s^2}$ . The above boost is the characteristic velocity of the turbulence caused by the wiggles, i.e.  $v_{\text{rot}} \simeq u$ . Hence, an estimate of the vorticity is

$$|\mathbf{w}| \simeq \frac{v_{\text{rot}}}{R} \simeq \frac{4\pi}{\Gamma t}, \quad (13)$$

where we used that the characteristic length-scale of the wiggles is given by

$$R \simeq \Gamma G \mu t, \quad (14)$$

with  $\Gamma$  determined by the rate of emission of gravitational radiation from the string, due to the oscillating wiggles. For gauge strings, simulations have shown that  $\Gamma \sim 100$ . For cosmic superstrings this may change somewhat because the intercommutation of the wiggles is suppressed and therefore one has less efficient loop formation and less kinks on the long string. Also, gravitational radiation may escape in the extra dimensions, though detailed simulations have yet to be performed [36].

Using the above, Eq. (13) suggests that the PMF generated at some time  $t_f$  is

$$B_f \sim \frac{m_p}{e} \frac{4\pi}{\Gamma t_f}. \quad (15)$$

Note that, remarkably,  $B_f$  does not depend on the value of  $G\mu$ .

Due to the high conductivity of the plasma the PMF is expected to freeze onto the plasma. The conservative approach, then, is to consider that the turbulent eddy is not gravitationally bound. This is reasonable to expect because cosmic string wake formation is no longer associated with structure formation, the latter occurring at overdensities generated due to inflation, which dominate the wake overdensities. For a non-gravitationally bound eddy one may estimate the strength of the magnetic field at galaxy formation by assuming that, being frozen into the plasma, the magnetic field conserves its flux and, therefore, scales as  $B \propto a^{-2}$ . Thus, scaling the above PMF down to the time of galaxy formation we obtain

$$B_{\text{gf}} \sim B_f \left( \frac{a_f}{a_{\text{gf}}} \right)^2 \sim \frac{4\pi m_p}{e \Gamma t_f} \left( \frac{t_f}{t_0} \right)^{4/3} (z_{\text{gf}} + 1)^2, \quad (16)$$

where  $t_0$  is the present time,  $z_{\text{gf}} \sim 6$  is the redshift at galaxy formation and we took into account that, until galaxy formation, the Universe remains matter dominated, i.e.  $a \propto t^{2/3}$ .

Similarly, we can find the coherence length of the magnetic field at galaxy formation. At formation the coherence scale is determined by the scale of the wiggles which stir the vortical motion [cf. Eq. (14)]

$$\ell_f \sim \Gamma G \mu t_f. \quad (17)$$

Since we consider an eddy which is not gravitationally bound  $\ell \propto a$ . Hence, at galaxy formation we find

$$\ell_{\text{gf}} \sim \left( \frac{a_{\text{gf}}}{a_f} \right) \ell_f \sim \frac{\Gamma G \mu t_f}{(z_{\text{gf}} + 1)} \left( \frac{t_0}{t_f} \right)^{2/3} \quad (18)$$

Eqs. (16) and (18) show that the dependence of the PMF strength and coherence on the time of formation  $t_f$  is very weak:

$$B_{\text{gf}}, \ell_{\text{gf}} \propto t_f^{1/3} \quad (19)$$

Indeed, it can be easily checked that, for  $t_f$  between recombination and galaxy formation, the variance of both these quantities is no more than an order of magnitude with the best results achieved when the PMF is generated at late times. Hence, adopting again a conservative approach we estimate  $B_{\text{gf}}$  and  $\ell_{\text{gf}}$  at the time of recombination  $t_{\text{rec}}$ .

After recombination there is some residual ionisation present in the plasma, which can allow the vortical generation of PMFs [35]. Setting  $t_f = t_{\text{rec}}$ , it is easy to find

$$B_{\text{gf}} \sim 10^{-23} \text{Gauss} \quad \text{and} \quad \ell_{\text{gf}} \sim 10^4 (G\mu) \text{Mpc}, \quad (20)$$

where we used  $v_s \gamma_s \sim 1$  and  $\Gamma \sim 100$ . If this PMF is carried by the plasma during the gravitational collapse of a galaxy, then flux conservation amplifies its strength by a factor

$$\left( \frac{\text{intergalactic distance at } t_{\text{gf}}}{\text{galactic size}} \right)^2 \sim \mathcal{O}(10^2)$$

while its coherence is decreased by a factor

$$\frac{\text{galactic size}}{\text{intergalactic distance at } t_{\text{gf}}} \sim \mathcal{O}(10^{-1}).$$

Hence, the seed field for the galactic dynamo is

$$B_{\text{seed}} \sim 10^{-21} \text{Gauss} \quad \text{and} \quad \ell_{\text{seed}} \sim 10^3 (G\mu) \text{Mpc}. \quad (21)$$

Comparing the above with the bound in Eq. (4) we see that such a seed field is strong enough to successfully trigger the dynamo and explain the galactic magnetic fields. However, the coherence requirements are more difficult to satisfy. Indeed, comparing the above to the bound in Eq. (5) we see that the latter can be satisfied only if  $G\mu \gtrsim 10^{-7}$ , which is in marginal conflict with the observations.

The situation can be somewhat improved if we consider PMF generation at much later times than recombination. The latest appropriate time corresponds to the epoch of earlier ionisation that precedes galaxy formation. Reionisation of the Universe at late times has indeed been detected by the WMAP, based on the observed decrease of the temperature angular power spectrum at high multipoles and by an excess in the TE cross-power spectrum on large angular scales, with respect to the case of no or little reionisation. Many believe that this reionisation occurs at two stages; late reionisation due to quasars at redshifts  $z_{\text{ri}} \simeq 6$  and earlier reionisation at redshifts of at least  $z_{\text{ri}} \gtrsim 15$  and up to (a few)  $\times 10$  possibly associated to Population III stars [37].

Taking  $z_f = z_{\text{ri}} \simeq 15$  it is easy to find that, due to Eq. (19), both strength and coherence of the seed field are intensified by an order of magnitude. However, generating the PMF that late implies that only a small fraction of the galaxies can benefit from the mechanism. This is because, even though the low intercommutation probability  $P$  results in a denser string network, one cannot envisage more than about  $t/\xi \leq P^{-1} \lesssim 10^3$  [cf. Eq. (3)] long

strings travelling at the comoving volume of the present horizon at the time of formation of the PMF. This means that magnetisation will appear in thin sheets of width given roughly by  $t_f^{-1}$ , which, for  $z_f = z_{\text{ri}}$  could be quite far apart (comoving distance:  $\sim 100$  Mpc), leaving a lot of the protogalaxies “untouched”. Also, since structure formation is not really related to string wakes, it is not certain how much of the magnetised plasma will find its way into galaxies (even though magnetised plasma dispenses more efficiently with angular momentum, which assists gravitational collapse). A fair portion of magnetised plasma will remain in the intergalactic medium and will result in intergalactic magnetic fields of order  $10^{-24}$  Gauss, which are far weaker than the ones observed [1] (the latter are thought to be expelled to the IGM by active galaxies through processes such as the Parker instability).

In contrast, a PMF generated just after recombination permeates most of the plasma because, at recombination the string network will be denser by a factor

$$\left(\frac{t_{\text{ri}}}{t_{\text{rec}}}\right) \times \left(\frac{a_{\text{rec}}}{a_{\text{ri}}}\right) \sim \left(\frac{t_{\text{ri}}}{t_{\text{rec}}}\right)^{1/3} \sim \mathcal{O}(10^2)$$

assuming it follows a scaling solution with  $P^{-\beta}$  strings per horizon. Hence the magnetised sheets could be as close as  $\sim 1$  Mpc comoving distance.

Still, stability arguments may inhibit the generation of a PMF for non-gravitationally bound eddies [34]. If this is so then we have to limit ourselves to gravitationally bound objects that have been captured by the rapidly oscillating wiggles, while the string traverses space. This would imply a much stronger PMF since there will be no further dilution due to the expansion of the Universe. However, this also means that not all the turbulent material in the string wake can be expected to become magnetised but only any preexisting lumps that have been caught by the passage of the string. Furthermore, since the dimensions of the magnetised region would not follow the expansion of the Universe, the coherence of the PMF would be much less than previously considered because  $\ell_{\text{gf}} \simeq \ell_f$ . This allows the possibility to satisfy the bound in Eq. (5) only if the PMF is generated rather late, i.e. at the reionisation time. However, as we have already mentioned, at late times the string wakes are far apart, which means that galaxies would only be sparsely magnetised.

### C. Vortical motions on inter-string distances

An alternative way for the generation of a PMF by a network of cosmic strings through the Harrison–Rees mechanism is considering vortical motions in the plasma stirred by travelling neighbouring strings in a string network. Travelling cosmic strings can also drag the plasma behind them. This causes circular motions over the inter-string distance (the separation between two neighbouring strings in the network) as neighbouring strings pass by one another in opposite directions.

One important aspect of the mechanism is the consideration of cosmic strings which also develop an *attractive* gravitational field, which assists to the drag of the plasma in the string trail. In order for this to occur we have to consider strings whose metric is slightly different compared to Eq. (10).

Let us consider a straight cosmic string, whose (2+1)-dimensional perpendicular spacetime is:

$$ds_{\perp}^2 = (1 - h_{00})[-dt^2 + dr^2 + (1 - \Delta/\pi)r^2 d\phi^2], \quad (22)$$

where  $h_{00}$  is the time-time component of a perturbation of the metric  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , with  $\eta_{\mu\nu}$  being the metric of Minkowski spacetime.  $h_{00}$  can be non-zero in models, where the effective energy per unit length  $\tilde{\mu}$  is different that the tension  $T$  of the string. From the above we see that spacetime remains conical, with a deficit angle  $\Delta$ . However, due to the  $(1 - h_{00})$  factor there is also some gravitational attractive force towards the string. This can be seen as follows.

The geodesic equation is,  $\frac{d^2 u^\lambda}{d\tau^2} + \Gamma_{\mu\nu}^\lambda u^\mu u^\nu = 0$ , where  $u^\mu = dx^\mu/d\tau = (1, \mathbf{v})$  is the 4-velocity and  $\Gamma_{\mu\nu}^\lambda \simeq \frac{1}{2}\eta^{\lambda\rho}(\partial_\nu h_{\mu\rho} + \partial_\mu h_{\nu\rho} - \partial_\rho h_{\mu\nu})$  are the Christoffel symbols. Since, for the plasma,  $|\mathbf{v}| \ll 1$ , the geodesic equation becomes

$$\frac{d^2 x^i}{d\tau^2} + \Gamma_{00}^i = 0, \quad (23)$$

where  $i$  denotes the spatial coordinates and  $\tau$  is the proper time. Since  $\Gamma_{00}^i = -\frac{1}{2}\partial_i h_{00}$  we find that the gravitational force per unit length is

$$\mathbf{f} = \frac{1}{2}\nabla h_{00}. \quad (24)$$

Now, let us investigate what this implies for the plasma particles when a string with such a gravitational field passes by. For this it is better to rewrite the metric in Eq. (22) in Cartesian coordinates

$$ds_{\perp}^2 = (1 - h_{00})(-dt^2 + dx_k dx^k), \quad (25)$$

where  $k = 1, 2$  and we need to extract from the above a wedge of deficit angle  $\Delta$ . Then, we have

$$d\tau^2 = -ds_{\perp}^2 = (1 - h_{00})dt^2(1 - \dot{x}_k \dot{x}^k). \quad (26)$$

Using  $h_{00}$  and also that  $\Gamma_{00}^i = -\frac{1}{2}\partial_i h_{00}$  we insert the above into Eq. (23) and obtain

$$2\ddot{x}^i = (1 - \dot{x}_k \dot{x}^k)\partial^i h_{00}, \quad (27)$$

with  $i = 1, 2$ . The above gives the acceleration felt by the particles due to the gravitational pull of the string, in the frame of the string.

Suppose that the string moves in the  $x$ -direction with constant velocity  $-v_s = -\sqrt{\dot{x}_k \dot{x}^k}$ . Then, for a particle in the position  $(x, y)$ , we have initially  $x = vt$  and  $y = \text{const}$ . The velocity boost felt by the particle towards the  $y$ -direction after its encounter with the string is

$$u_y = \int_{-\infty}^{\infty} \ddot{y} dt = \frac{1}{2v_s \gamma_s^2} \int_{-\infty}^{\infty} \partial_y h_{00} dx, \quad (28)$$

where we also used Eq. (27). Taking into account the deficit angle, the relative boost between the particles on the opposite sides of the string is  $\delta u_y = v_s \Delta + 2|u_y|$ . Switching to the particle frame gives

$$u = \gamma_s \delta u_y = \Delta v_s \gamma_s + \frac{2I}{v_s \gamma_s}, \quad (29)$$

where

$$I \equiv \int_{-\infty}^{\infty} f_y dx, \quad (30)$$

and we also considered Eq. (24). In Eq. (29) the first term is due to the conical spacetime [cf. Eqs. (11) and (12)], while the second term is due to the gravitational attractive force.

The deflection of particles in the string spacetime results in a net drag of the plasma behind the string. This is due to the fact that the magnitude of the particle velocity is not modified after the interaction with the string. The velocity of plasma dragging can be estimated by Taylor expanding the particle velocity given in Eq. (29). To the lowest order we find

$$\delta v \simeq \frac{1}{2} \frac{u^2}{v_s}. \quad (31)$$

The backreaction of this effect is a decelerating force on the string, which can be estimated as follows

$$f_{\text{drag}} = \int \frac{d^2 p}{dt dz} dx dy \simeq 2R \rho v_s \delta v, \quad (32)$$

where  $f_{\text{drag}}$  is the drag force per unit length,  $p \sim \rho v_s$  is the momentum of the plasma in the string frame,  $dx \simeq (\delta v) dt$  is the drag of the plasma and  $R$  is the inter-string distance (over which  $dy$  is integrated).

A string segment of length  $R$  may transfer momentum to the plasma in the inter-string volume  $\sim R^3$ . In this way the string network can induce vortical motions to the plasma on inter-string scales. The total force on a plasma volume of dimensions comparable to the inter-string distance  $R$  is

$$F \simeq \int_0^R f_{\text{drag}} dz \sim R^2 \rho u^2, \quad (33)$$

where we also used Eq. (31). Hence, the typical rotational velocity is estimated as

$$FR \simeq \frac{1}{2} M_R v_{\text{rot}}^2 \Rightarrow v_{\text{rot}} \sim \frac{\sqrt{F/\rho}}{R} \sim u, \quad (34)$$

where  $M_R \sim \rho R^3$  is the mass in the inter-string volume and we used Eq. (33).

The strength of the PMF generated by the vortical motions can be estimated using Eqs. (8) and (9), with  $v_{\text{rot}} \sim u$ . It is straightforward to obtain

$$B \simeq \frac{m_p}{e} \frac{v_{\text{rot}}}{R} \Rightarrow B_f \sim \frac{m_p}{e} \frac{\gamma_s}{P^\beta t_f} \left[ \Delta + \frac{2I}{(v_s \gamma_s)^2} \right], \quad (35)$$

where  $R \sim P^\beta(v_s t)$ . Obviously, the coherence of the PMF is given by the inter-string distance, i.e.

$$\ell_f \sim P^\beta v_s t_f. \quad (36)$$

In contrast to the previous case, we will concentrate on gravitationally bound eddies, which do not suffer from stability problems. The reason is that, since the total of inter-string volumes spans all space (in contrast to wakes behind the moving strings), it follows that all the gravitationally bound objects lie inside volumes that may well be rotated by the string network. From Eq. (36) we see that the inter-string distance can be much smaller than the horizon and, therefore, overdensities that become causally connected, collapse and detach from Hubble expansion are quite likely to be affected by strings. We expect the most prominent PMF generation to occur near recombination, when the plasma is still substantially ionised and just after structure formation begins.

In the following we will estimate the strength and coherence of the inter-string PMF in two cases, which can be described by a metric of the form shown in Eq. (22).

### 1. Wiggly strings

PMF generation on inter-string distances by a network of cosmic strings was first considered by Avelino and Shellard [11]. They considered the case of wiggly strings, which can develop an attractive gravitational field because their effective energy per unit length  $\tilde{\mu}$  and their tension  $T$  are different (whereas for a straight string  $T = \mu$ ). This is due to coarse-graining the small scale structure (wiggles) on the string. The relation between  $\tilde{\mu}$  and  $T$  is

$$\tilde{\mu} T = \mu^2. \quad (37)$$

Simulations for gauge strings estimate  $\tilde{\mu} \approx 1.6\mu$ , which means that  $\tilde{\mu} - T \approx 0.6\mu$ .

For wiggly strings it has been found that [8]

$$h_{00} = -4G(\tilde{\mu} - T) \ln(r/r_0), \quad (38)$$

where  $r_0$  is the radius of the string core. Inserting this into Eq. (24) we find an attractive force per unit length

$$f = -\frac{2G(\tilde{\mu} - T)}{r}. \quad (39)$$

Hence, the total boost is

$$u = 8\pi G \tilde{\mu} v_s \gamma_s + \frac{4\pi G(\tilde{\mu} - T)}{v_s \gamma_s}, \quad (40)$$

where we used Eqs. (12) and (30) considering also that the deficit angle is still given by Eqs. (11) with  $\mu \rightarrow \tilde{\mu}$ . Using the above, Eq. (35) suggests

$$B_f \sim \frac{m_p}{e} \frac{8\pi G \tilde{\mu} \gamma_s}{P^\beta t_f} \left[ 1 + \frac{\tilde{\mu} - T}{2\tilde{\mu}(v_s \gamma_s)^2} \right]. \quad (41)$$

Evaluating the above at recombination and considering also that the gravitational collapse of a galaxy amplifies the PMF by a factor of  $\mathcal{O}(10^2)$  we find

$$B_{\text{seed}} \sim 10^{-15} P^{-\beta} (G\mu) \text{ Gauss}, \quad (42)$$

where we considered  $v_s \gamma_s \sim 1$ . Comparing the above with the bound in Eq. (4) we see that the generated PMF is strong enough to seed the galactic dynamo provided

$$G\mu \geq 10^{-15} P^\beta, \quad (43)$$

which is satisfied for the entire range of  $G\mu$  shown in Eq. (2).

With respect to coherence, evaluating Eq. (36) at recombination and considering also that galactic gravitational collapse reduces the coherence of a PMF by a factor of  $\mathcal{O}(10^{-1})$ , we find

$$\ell_{\text{seed}} \sim 10^{-2} P^\beta \text{ Mpc}. \quad (44)$$

Comparing this with the bound in Eq. (5) we see that the PMF is coherent enough for the dynamo, provided

$$P^\beta \geq 10^{-2}. \quad (45)$$

From Eqs. (35) and (36) it is evident that if we consider later times for the PMF generation, the coherence of the seed field is improved but its strength is diluted.

The above show that a network of wiggly cosmic superstrings may well be responsible for the galactic magnetic fields even though  $G\mu$  is small enough not to dominate structure formation.

## 2. Superconducting strings

Another realisation of a PMF generation over inter-string distances was investigated by one of us (KD) considering a network of superconducting cosmic strings [12]. Since D-strings arise in supersymmetric theories then they will have fermion zero modes in the string core [22]. It was shown in [22] that some zero modes survive supersymmetry breaking, depending on the details of the breaking mechanisms. Consequently, cosmic superstrings can also develop currents. If the current is electromagnetically coupled, then they could be superconducting.

As in wiggly strings, superconducting strings have different  $\tilde{\mu}$  and  $T$ . Hence, they too generate an attractive gravitational field. Indeed, as shown in Ref. [12], in this case we have

$$h_{00} = -4G[J^2 + (\tilde{\mu} - T)] \ln(r/r_0) - 4GJ^2 [\ln(r/r_0)]^2, \quad (46)$$

where  $J$  is the string current and

$$\tilde{\mu} \simeq \mu + \frac{J^2}{4Ke^2} \quad \text{and} \quad T \simeq \mu - \frac{J^2}{4Ke^2} \quad (47)$$

with  $e$  being the charge of the current carriers and  $K \gtrsim 1$  is a constant depending on the underlying model. Using

the above in Eq. (24) we obtain the attractive force per unit length

$$f = -\frac{2GJ^2}{r} \left[ 1 + \frac{\tilde{\mu} - T}{J^2} + 2 \ln(r/r_0) \right]. \quad (48)$$

The deficit angle this time is given by [12]

$$\Delta = 8\pi G \left\{ \tilde{\mu} + J^2 \left[ \frac{1}{2} + \ln(r/r_0) \right] \right\} \quad (49)$$

Hence, the total boost is

$$u = 8\pi G\mu v_s \gamma_s + 4\pi G(QJ)^2 \left( v_s \gamma_s + \frac{1}{v_s \gamma_s} \right), \quad (50)$$

where  $Q \sim \mathcal{O}(10)$  is a constant (associated with the string radius) due to the self-inductance of the string. Using the above, Eq. (35) gives

$$B_f \sim \frac{m_p}{e} \frac{8\pi G\mu \gamma_s}{P^\beta t_f} \left\{ 1 + \frac{(QJ)^2}{2\mu} \left[ 1 + \frac{1}{(v_s \gamma_s)^2} \right] \right\}. \quad (51)$$

If the string velocity is relativistic then the first term in the curly brackets dominates the right-hand-side of the above. This is because the string current is bounded from above as

$$J \leq J_{\text{max}} \equiv e\sqrt{\mu}. \quad (52)$$

Hence, if  $v_s \gamma_s \sim 1$  the generated PMF does not differ in strength and coherence from the wiggly string case, resulting in a seed field with the characteristics shown in Eqs. (42) and (44).

However, there is a chance that one may generate a much stronger PMF. Indeed, in an earlier work of ours, we have shown that, if the strings carry electrically charged currents, excessive friction between the strings and the plasma can result in strong damping of the string motion [38]. The reason is that a charged current carrying string is surrounded by a Biot-Savart magnetic shield, which encloses the string in a magnetocylinder, similar to the Earth's magnetosphere. The magnetocylinder is impenetrable by the ionised plasma, which is deflected away from the path of the moving string, resulting in a friction force, which may heavily damp the string motion. In Ref. [38] we have studied the dynamics and the evolution of such a string network. We have found that the strings reach a terminal velocity  $v_T$  given by

$$v_T^2 \sim \frac{G\mu}{\sqrt{GJ^2}}, \quad (53)$$

which could be rather small  $v_T \ll 1$  if the string current is large enough. As we have shown in Ref. [38], the string network in this case does reach a scaling solution (fixed number of strings per horizon volume), which, however, is much denser than the usual case. In the case of cosmic superstrings the density of the string network would be further increased by a factor  $P^{-\beta}$  due to the small intercommutation probability.

Using the estimate of the terminal velocity in Eq. (53), it can be easily checked that, if the string current is

$$J > J_* \equiv e^{-1}(G\mu)^{1/6} J_{\max}, \quad (54)$$

then the last term on the right-hand-side of Eq. (51) is the dominant <sup>2</sup>. Then the generated PMF is

$$B_f \sim \frac{4\pi Q^2 \sqrt{e} m_p}{P^{\beta} t_f} \sqrt{G\mu} \left( \frac{J}{J_{\max}} \right)^3, \quad (55)$$

where we used Eqs. (51), (52) and (53). Evaluating the above at recombination and considering the amplification of order  $\mathcal{O}(10^2)$  due to the gravitational collapse of the galaxy, we find

$$B_{\text{seed}} \sim 10^{-14} P^{-\beta} \sqrt{G\mu} (J/J_{\max})^3 \text{ Gauss} \quad (56)$$

which can be quite sufficient for the dynamo for  $J_* < J \lesssim J_{\max}$ .

Now, the coherence of this PMF is determined by the inter-string distance, i.e.

$$\ell_f \sim P^{\beta} v_T t_f \sim \frac{P^{\beta} t_f}{\sqrt{e}} (G\mu)^{1/4} \left( \frac{J_{\max}}{J} \right)^{1/2}, \quad (57)$$

where we used Eqs. (52) and (53). Evaluating again at recombination and also considering that galactic gravitational collapse reduces the coherence of the PMF by  $\mathcal{O}(10^{-1})$ , we obtain

$$\ell_{\text{seed}} \sim 10^{-4} P^{\beta} (G\mu)^{1/4} \sqrt{J_{\max}/J} \text{ Mpc}, \quad (58)$$

which only marginally satisfies the bound in Eq. (5).

Hence, from the above, we see that superconducting strings can result in strong and coherent magnetic fields which can trigger the galactic dynamo and explain thereby the observed galactic magnetic fields.

#### IV. CONCLUSIONS

We have investigated the generation of a Primordial Magnetic Field (PMF) in the Universe, through the effect of a network of cosmic superstrings onto ionised plasma, after recombination. The PMF is created by the Harrison–Rees mechanism, which considers a spinning volume of ionised plasma, in which the electron and ion fluids are spinning with different angular velocity. Hence, circular currents arise that give birth to a PMF.

<sup>2</sup> This is the term due to the gravitational attraction.

The network of cosmic superstrings can cause such vortical motions in the plasma, due to the gravitational effects of moving string segments. There are two possibilities; one being vortical motions inside string wakes, caused by the rapidly oscillating wiggles on the strings, and the other being vortical motions over inter-string distances caused by the relative motion of travelling neighbouring strings of the network. The latter effect can be further intensified if the strings exert an attractive gravitational field onto surrounding matter, as is the case with wiggly or superconducting strings. In our work we have focused on the possibility that such a PMF can be sufficiently strong and coherent to seed the galactic dynamo mechanism and explain the observed galactic magnetic fields.

We have studied all the above cases and found that it is always possible to create a PMF strong enough to seed the galactic dynamo. However, achieving the required coherence for this field is more challenging. Indeed, for PMF generation due to string wiggles, we have found that a seed field as coherent as  $\ell_{\text{seed}} \sim 100 \text{ pc}$  can be created only if the string tension assumes its highest possible value  $G\mu \sim 10^{-7}$ . This constraint is somewhat relaxed if the PMF is generated as late as the late reionisation period  $z_{\text{ri}} \sim 15$ , but, in this case, only a fraction of the galaxies is expected to be magnetised, because the string wakes are far apart. In the case of inter-string PMF generation, coherence is easier to attain. Indeed, a coherent enough seed field can be obtained provided the inter-commutation probability is not extremely low. Cosmic superstrings that carry substantial currents may generate a really intense PMF (up to  $B_{\text{seed}} \sim 10^{-15} \text{ Gauss}$ ) but at the expense of its coherence. An adequately coherent field, in this case, also requires a high value for  $G\mu$ .

Cosmic superstrings are a probable result of brane cosmology and brane inflation models. However, since they are not the primary cause of the acoustic peaks in CMB spectrum, such strings appeared to have limited observational signatures. As a result, their observable consequences were thought to be limited to gravitational lensing events. In this paper we show that cosmic superstrings may have further important cosmological consequences. In particular, they may be responsible for the observed galactic magnetic fields and, in general, long range magnetic fields in the IGM. Since the stellar magnetic dynamo is thought to be triggered by the galactic magnetic field, it seems plausible that cosmic superstrings could be the principle source of magnetisation in the Universe.

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- [1] P. P. Kronberg, Rept. Prog. Phys. **57** (1994) 325.
  - [2] R. Beck, A. Brandenburg, D. Moss, A. Shukurov and D. Sokoloff, Ann. Rev. Astron. Astrophys. **34** (1996) 155.
  - [3] K. Enqvist and P. Olesen, Phys. Lett. B **319** (1993) 178;

- A. C. Davis and K. Dimopoulos, Phys. Rev. D **55** (1997) 7398; K. Dimopoulos and A. C. Davis, Phys. Lett. B **390** (1997) 87.
- [4] A. Brandenburg, K. Enqvist and P. Olesen, Phys. Rev.



- D **54** (1996) 1291.
- [5] M. S. Turner and L. M. Widrow, Phys. Rev. D **37** (1988) 2743.
  - [6] K. Dimopoulos, T. Prokopec, O. Tornkvist and A. C. Davis, Phys. Rev. D **65** (2002) 063505; A. C. Davis, K. Dimopoulos, T. Prokopec and O. Tornkvist, Phys. Lett. B **501** (2001) 165.
  - [7] D. Grasso and H. R. Rubinstein, Phys. Rept. **348** (2001) 163; M. Giovannini, Int. J. Mod. Phys. D **13** (2004) 391.
  - [8] T. Vachaspati and A. Vilenkin, Phys. Rev. Lett. **67** (1991) 1057.
  - [9] T. Vachaspati, Phys. Rev. D **45** (1992) 3487.
  - [10] D. N. Vollick, Phys. Rev. D **48** (1993) 3585.
  - [11] P. P. Avelino and E. P. S. Shellard, Phys. Rev. D **51** (1995) 5946.
  - [12] K. Dimopoulos, Phys. Rev. D **57** (1998) 4629.
  - [13] S. Sarangi and S.-H. H. Tye, Phys. Lett. B **536** (2002) 185. N. Jones, H. Stoica and S.-H. H. Tye, JHEP **07** (2002) 051.
  - [14] E.J. Copeland, R.C. Myers and J. Polchinski, JHEP **06** (2004) 013.
  - [15] J. Polchinski, hep-th/0412244.
  - [16] A. C. Davis and T. W. B. Kibble Contemporary Physics (to appear), hep-th/0505050
  - [17] S.C. Davis, P. Binetruy and A.C. Davis, Phys. Lett. B **611** (2005) 39.
  - [18] T.W.B. Kibble, astro-ph/0410073.
  - [19] E. Jeong and G. F. Smoot astro-ph/0406432
  - [20] S. C. Davis, A. C. Davis and M. Trodden, Phys. Lett. B **405** (1997) 257.
  - [21] Dvali, G., Kallosh, R. & van Proeyen, A., JHEP **0401** (2004) 035
  - [22] S. C. Davis, A. C. Davis and M. Trodden, Phys. Rev. D **57** (1998) 5184.
  - [23] G. Dvali and A. Vilenkin. JCAP **0403** (2004) 010
  - [24] M. Sakellariadou, hep-th/0410234.
  - [25] T. Damour and A. Vilenkin Phys. Rev. Lett. **78** (1997) 2288.
  - [26] E. Babichev and M. Kachelriess, Phys. Lett. B **614** (2005) 1.
  - [27] A. Avgoustidis and E. P. S. Shellard, hep-ph/0410349
  - [28] J. Urrestilla, A. Achucarro and A.C. Davis, Phys. Rev. Lett. **92** (2004) 251302.
  - [29] P. Binetruy, G. Dvali, R. Kallosh and A. van Proeyen, Class. Quant. Grav. **21** (2004) 3137.
  - [30] T. Vachaspati and A. Achucarro, Phys. Rev. D **44** (1991) 3067.
  - [31] A. C. Davis, M. Lilley and O. Tornkvist, Phys. Rev. D **60** (1999) 021301.
  - [32] R. M. Kulsrud and S. W. Anderson, Astrophys. J. **396** (1992) 606.
  - [33] E.R. Harrison, Nature **224** (1969) 1089; Mon. Not. R. Astr. Soc. **147** (1970) 279; Phys. Rev. Lett. **30** (1973) 188.
  - [34] M. J. Rees, Quart. Jl. R. Astron. Soc. **28** (1987) 197.
  - [35] I. N. Mishustin and A. A. Ruzmaikin, Zh. Eksp. Teor. Fiz. **61** (1971) 441; Sov. Phys. JETP **34** (1972) 233.
  - [36] T. Damour and A. Vilenkin, Phys. Rev. D **71** (2005) 063510.
  - [37] A. Kogut *et al.*, Astrophys. J. Suppl. **148** (2003) 161; A. Kogut, astro-ph/0306048.
  - [38] K. Dimopoulos and A. C. Davis, Phys. Rev. D **57** (1998) 692; Phys. Lett. B **446** (1999) 238.